

7th Math

Compound Interest

Interest can be calculated in two different ways: simple and compound. Last section you learned to calculate interest using the **simple interest** formula: $I = Prt$. This formula calculates the interest only. The new amount (A) can be determined by adding the original amount/principle (P) to the interest earned (I): $A = P + I$. Simple interest is a calculation based on the original principle only. Over time, the amount of interest earned is year is the same. For example, an investment earns 4% annual interest. Each year, a \$1000 investment will earn \$40. After the first year the total is \$1040, the second year total is \$1080, and the third year is \$1120.

Compound interest is the common method of calculating interest. While it is more common, it is also a more challenging computation. In compound interest, your interest earns interest. This means that each year your interest is calculated on the new total, not just the original amount.

Example A: The comparison of an investment of \$1000 at 4% annual interest for 5 years, showing the annual totals.

	Simple Interest	Compound Interest
Year 1	Interest: $1000(0.04)(1) = \$40$ Total: $1000 + 40 = \$1040$	Interest: $1000(0.04)(1) = \$40$ Total: $1000 + 40 = \$1040$
Year 2	Interest: $1000(0.04)(2) = \$80$ Total: $1000 + 80 = \$1080$	Interest: $1040(0.04)(1) = \$41.60$ Total: $1040 + 41.60 = \$1081.60$
Year 3	Interest: $1000(0.04)(3) = \$120$ Total: $1000 + 120 = \$1120$	Interest: $1081.60(0.04)(1) = \$43.26$ Total: $1081.60 + 43.26 = \$1124.86$
Year 4	Interest: $1000(0.04)(4) = \$160$ Total: $1000 + 160 = \$1160$	Interest: $1124.86(0.04)(1) = \$44.99$ Total: $1124.86 + 44.99 = \$1169.85$
Year 5	Interest: $1000(0.04)(5) = \$200$ Total: $1000 + 200 = \$1200$	Interest: $1169.85(0.04)(1) = \$46.79$ Total: $1169.85 + 46.79 = \$1216.64$

According to the example, simple interest is always based on the original amount/principle. Therefore the interest earned each year is constant: \$40. However, in compound interest, the interest is calculated on the new total amount of money each year. After the first year, both simple and compound interest are the same: \$1040. However in the second year, compound interest is based on the new total, \$1040, whereas simple interest is still calculated on the original \$1000. By the end of five year, the compound interest calculation has earned \$16.64 more than the simple interest. Over time, this difference will become greater and greater.

The following is the formula for **yearly compound interest**.

$$A = P(1 + r)^t$$

Where **A** is the total new total amount of money after an initial amount (**P**) is earning **r** annual percent for **t** years. The annual percentage rate, **r**, is in decimal form.

Example B: \$5000 is deposited into a savings account earning 2.86%. How much is the savings account worth if left untouched for 4 years, if the interest is compounded yearly?

Use the yearly compound interest formula: $A = P(1 + r)^t$.

$P = \$5000$, $r = 0.0286$, and $t = 4$. After substitution:

$$A = 5000(1 + 0.0286)^4 \quad \text{Follow the order of operations}$$

$$A = 5000(1.0286)^4 \quad \text{Do not round}$$

$$A = 5000(1.119402004) \quad \text{Do not round}$$

$$A = 5597.010018 \quad \text{Rounding to the nearest cent}$$

$$A = \$5597.01$$

The savings account is worth \$5597.01 after 4 years.

A calculator must be used to answer compound interest questions. To calculate $(1.0286)^4$, either use a calculator to multiply: $(1.0286)(1.0286)(1.0286)(1.0286)$, or use the exponent button on your calculator. The exponent button looks like: y^x or \wedge

To input $(1.0286)^4$ do the following: $1.0286 \ y^x \ 4$ or $1.0286 \ \wedge \ 4$.

Example C: What is the interest earned after 2 years if \$2500 is deposited into a certificate of deposit earning 3.4% annual interest?

$$A = 2500(1 + 0.034)^2$$

$$A = 2500(1.034)^2 \quad \text{calculator sequence: } 2500 \ \boxed{\times} \ 1.034 \ \boxed{\wedge} \ 2 \ \boxed{\text{ENTER}}$$

$$A = \$2672.89$$

The balance after 2 years is \$2672.89.

Interest earned is total amount, A, less the principle, P ($I = A - P$).

This means that \$172.89 has been earned in interest. ($I = 2672.89 - 2500$)

~~~~Practice and Problem Solving~~~~

You may do all work on this page. Use a calculator, showing steps.

Determine the **balance** (A) and the amount of **interest** (I) earned for the following questions, when:

a. the account is earning simple interest ( $I = Prt$ ,  $A = P + I$ )

b. the account is earning yearly compounded interest ( $A = P(1 + r)^t$ ,  $I = A - P$ )

1. \$3000 is invested at 3% annual interest for 3 years.

a. Simple Interest

b. Compound Interest

2. \$1234 is invested at 5.6% annual interest for 7 years.

a. Simple Interest

b. Compound Interest

3. \$10,000 is invested at 0.75% annual interest for 2 years.

a. Simple Interest

b. Compound Interest

4. \$987.65 is invested at 0.4% annual interest for 5 years.

a. Simple Interest

b. Compound Interest

5. \$500 is invested at 4.3% annual interest for 18 months.

a. Simple Interest

b. Compound Interest

6. \$1000 is invested at 2.9% annual interest for 6 months.

a. Simple Interest

b. Compound Interest