Algebra ***Dividing, Factors, and Monomial Factoring***

Objectives: To divide polynomials by monomials and to find monomial factors of polynomials.

Factors are the number(s) used to multiply to make a larger number. For example, the factors of 24 are: 1, 2, 3, 4, 6, 12, and 24. For our purposes, factors are always integers, therefore no fractions or decimals. When we discuss factoring, we often refer to the prime factorization of a number. Prime factorizations look at all the prime numbers that are factors of the whole. These prime factorizations can be done on variable expressions as well (i.e. $12x^2y^3 = 2\cdot 2\cdot 3\cdot x\cdot x\cdot y\cdot y\cdot y$).

If you know one factor of a number, the other factor can be determined.

Example 1

Fill in the blank with the missing factor

$96 = 12(\_\_\_\_)$

$96 = 12(8)$

Example 2

Fill in the blank with the missing factor.

$36x^8 = 3x^3(\_\_\_\_)$

$36x^8 = 3x^3(12x^5)$

Example 3

Fill in the blank with the missing factor.

$48x^5y^4z^3 = 6x^2y^3z\ (\_\_\_\_)$

$48x^5y^4z^3 = 6x^2y^3z\ (8x^3yz^2)$

Find the missing factor.

1. $6t^4 = 2t(\_\_\_\_\_)$
2. $12w^6 = 3w^2(\_\_\_\_\_)$
3. $9a^3b^4 = 3a^2b^2(\_\_\_\_\_)$
4. $18pq^3 = 6pq(\_\_\_\_\_)$
5. $-35x^3y^5 = 7x^2y(\_\_\_\_\_)$
6. $48c^5d^4 = -3c^3d^2(\_\_\_\_\_)$

Before we can monomially factor, we must review the concept of GCF (greatest common factor). In factoring a monomial, we always look to factor out the greatest common factor. To determine the GCF of algebraic terms focus on the number then the variable portion. Keep in mind that the GCF can be no bigger than the smaller factor.

Example 4

Find the GCF of 48 and 60.

The largest number that goes into 48 and 60 is 12.

The GCF is 12.

Example 5

Find the GCF of $48x^6$ and $60x^4$

The GCF of 48 and 60 is 12.

$x^6 = xxxxxx$ and $x^4 = xxxx$, they both have $xxxx = x^4$ in common.

So the GCF = $12x^4$

Example 6

Find the GCF of $48x^6y^3z^5$ and $60x^4y^4z^3$

The GCF is $12x^2y^3z^3$

In general, take the variable with the lesser exponent.

Find the GCF of each set of monomials.

7. $3x^2, 9x^3$
8. p^2q^3, p^3q^2
9. $20ax^3, 30abx$
10. $48a^2bc^3, 72ab^3c^2$
11. $36x^2y^2z^2, 24xy^2z^3$
12. $25p^2q^3, 15p^2q^2, 35pq^4$

This all culminates in our ability to monomially factor polynomials. Factor is the opposite of distributing/FOILing. In the last unit, we used the distributive property to simplify expressions. Now we use factoring to undo that process.

In the past…simplify: $3x(2x+5) = 6x^2 + 15x$. Now the present…factor: $5x^2 + 10x = 5x(x+2)$.

Example 7

Factor: $12x - 18$

Find the GCF of $12x$ and $18 = 6$

$12x - 18 = 6(2x - 3)$

Example 8

Factor: $24mn + 16n$

Find the GCF of $24mn$ and $16n = 8n$

$24mn + 16n = 8n(3m + 2)$

Example 9

Factor: $77r^7s^7 + 84r^8s^4$

Find the GCF of

$77r^7s^7$ and $84r^8s^4 = 11r^7s^4$

$77r^7s^7 + 84r^8s^4 = 11r^7s^4(7s^3 + 12r)$

Factor.

13. $15a - 25b + 20$
14. $6x^2 + 10x$
15. $7y^3 - 21y^2 - 14y$
16. $4x^5 - 6x^3 + 14x$
17. $6ab^2 - 8a^2b$
18. $16x^3y + 24x^4y^3$
19. $48a^3b^2 - 72a^2b^3$
20. $14m^3n^3 - 21m^2n^2 + 35mn$
21. $6x^4 - 9x^3y + 12x^2y^2$